Solutions

EE 362: Signals and Systems Name:----------

Spring 2019

Homework #11 (Due: Tuesday, April 23, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

1. Compute Laplace transform of the following functions. [8 points]

a) $x(t) = \begin{cases} 2, & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

b) $x(t) = 2 - 4e^{-2t}$

c) $x(t) = te^{-2t}\cos(3t)$

d) $x(t) = (1 - e^{-2t})^2$

e) $x(t) = \cos^2(3t)$

2. Compute the initial values and final values (if possible) of the following functions. [4 points]

a) $X(s) = \frac{2}{s^4+6s^3+11s^2+6s}$

b) $X(s) = \frac{10s^2}{(s+1)(s^2+1)}$

3. Compute the inverse Laplace transform of the following functions. [10 points]

a) $X(s) = \frac{2}{s^2+3s+2}$

b) $X(s) = \frac{3s+5}{s^2+7}$

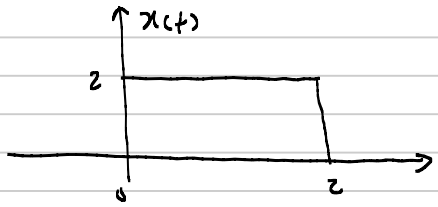
c) $X(s) = \frac{5}{s^2-9}$

d) $X(s) = \frac{3(s+3)}{s^2+6s+8}$

e) $X(s) = \frac{s+1}{s^4+4s^3+4s^2}$

Note: Problems must be solved by hand. However, you can use MATLAB commands such as roots, residue, laplace, and ilaplace to help in verifying your answers.

(1) a) $x(t) = \begin{cases} 2, & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$



$\Rightarrow x(t) = 2u(t) - 2u(t-2)$

$\therefore X(s) = \frac{2}{s} - \frac{2}{s} e^{-2s}$

b) $x(t) = 2 - 4e^{-2t}$

$\therefore X(s) = \frac{2}{s} - \frac{4}{s+2}$

c) $x(t) = t e^{-2t} \cos(3t)$

$\cos(3t) \leftrightarrow \frac{s}{s+9}$

$\therefore e^{-2t} \cos(3t) \leftrightarrow \frac{s+2}{(s+2)^2+9}$

$\therefore X(s) = \left[-\frac{d}{ds} X(s)\right] = -\frac{d}{ds}\left(\frac{s+2}{(s+2)^2+9}\right) = -\frac{(s^2+4s+13) - (s+2)(2s+4)}{(s^2+4s+13)^2}$

$= -\frac{-s^2-2s+5}{(s^2+4s+13)^2} = \frac{s^2+2s-5}{(s^2+4s+13)^2}$

d) $x(t) = (1-e^{-2t})^2$

$(1-e^{-2t})^2 = (1-e^{-2t})(1-e^{-2t}) = 1 - 2e^{-et} + e^{-4t}$

$\therefore X(s) = \frac{1}{s} - \frac{2}{s+2} + \frac{1}{s+4}$

e) $x(t) = \cos^2(3t)$

$\therefore x(t) = \frac{1}{2} + \frac{1}{2}\cos(6t)$

$\therefore X(s) = \frac{1}{2s} + \frac{s}{2(s^2+36)}$

(2)

a) $x(0) = \lim_{s\to\infty} [s\,X(s)] = \left.\frac{2}{s^3+6s^2+11s+6}\right|_{s\to\infty} = 0$

$x(\infty) = \lim_{s\to 0} [s\,X(s)] = \left.\frac{2}{s^3+6s^2+11s+6}\right|_{s\to 0} = \frac{1}{3}$

b) $x(0) = \lim_{s\to\infty} [sX(s)] = \left.\frac{10\,s^3}{(s+1)(s^2+1)}\right|_{s\to\infty}$

$= \left.\frac{10\,s^3}{s^3+s^2+s+1}\right|_{s\to\infty} = \left.\frac{10}{1+1/s+1/s^2+1/s^3}\right|_{s\to\infty} = 10$

The final value theorem is not applicable because $X(s)$ has two poles on the imaginary axis.

(3)

a) $X(s) = \frac{2}{s^2+3s+2} = \frac{2}{(s+2)(s+1)}$

$X(s) = \frac{C_1}{s+2} + \frac{C_2}{s+1}$

$C_1 = [(s+2)X(s)]_{s=-2} = -2$

$C_2 = [(s+1)X(s)]_{s=-1} = 2$

$\therefore X(s) = \frac{2}{s+1} - \frac{2}{s+2}$

$\therefore x(t) = 2e^{-t} - 2e^{-2t}, \quad t \geq 0$

b)

$X(s) = \frac{3s+5}{s^2+7} = \frac{3s}{s^2+7} + \frac{5}{s^2+7}$

$= 3\left(\frac{s}{s^2+7}\right) + \frac{5}{\sqrt{7}}\left(\frac{\sqrt{7}}{s^2+7}\right)$

$\therefore x(t) = 3\cos(\sqrt{7}t) + \frac{5}{\sqrt{7}}\sin(\sqrt{7}t), \quad t \geq 0$

c) $X(s) = \frac{5}{s^2-9} = \frac{5}{(s+3)(s-3)} = \frac{C_1}{s+3} + \frac{C_2}{s-3}$

$$c_1 = \left[(s+3)X(s)\right]_{s=-3} = -\frac{5}{6}$$

$$c_2 = \left[(s-3)X(s)\right]_{s=3} = \frac{5}{6}$$

$$\therefore X(s) = \frac{5/6}{s-3} - \frac{5/6}{s+3}$$

$$\therefore x(t) = \frac{5}{6}e^{3t} - \frac{5}{6}e^{-3t}, \quad t \geq 0$$

d) $$X(s) = \frac{3(s+3)}{s^2+6s+8} = \frac{3(s+3)}{(s+4)(s+2)} = \frac{c_1}{s+4} + \frac{c_2}{s+2}$$

$$c_1 = \left[(s+4)X(s)\right]_{s=-4} = 1.5$$

$$c_2 = \left[(s+2)X(s)\right]_{s=-2} = 1.5$$

$$\therefore X(s) = \frac{1.5}{s+4} + \frac{1.5}{s+2}$$

$$\therefore x(t) = 1.5e^{-4t} + 1.5e^{-2t}, \quad t \geq 0$$

e) $$X(s) = \frac{s+1}{s^4+4s^3+4s^2} = \frac{s+1}{s^2(s^2+4s+4)} = \frac{s+1}{s^2(s+2)^2}$$

$$X(s) = \frac{c_1}{s^2} + \frac{c_2}{s} + \frac{c_3}{(s+2)^2} + \frac{c_4}{(s+2)}$$

$$c_1 = \left[s^2X(s)\right]_{s=0} = \frac{1}{4}$$

$$c_2 = \left[\frac{d}{ds}\left(s^2X(s)\right)\right]_{s=0} = \frac{(s+2)^2 - (s+1)(2(s+2))}{(s+2)^4}\Bigg|_{s=0} = 0$$

$$c_3 = \left[(s+2)^2X(s)\right]_{s=-2} = \frac{s+1}{s^2}\Bigg|_{s=-2} = -\frac{1}{4}$$

$$c_4 = \left[\frac{d}{ds}\left((s+2)^2X(s)\right)\right]_{s=-2} = 0 \quad \Rightarrow X(s) = \frac{1/4}{s^2} - \frac{1/4}{(s+2)^2}$$

$$\therefore x(t) = \frac{1}{4}t - \frac{1}{4}te^{-2t}, \quad t \geq 0$$

MATLAB

1-a

```
syms x X x1
x1=2*heaviside(x)-2*heaviside(x-2);
X=laplace(x1)
```

```
ans: 2/s - (2*exp(-2*s))/s
```

1- b

```
syms x t X
x=2-4*exp(-2*t);
X=laplace(x)
```

```
ans: 2/s - 4/(s + 2)
```

1- c

```
syms x t X
x=t*exp(-2*t)*cos(3*t);
X=laplace(x)
```

```
ans: ((2*s + 4)*(s + 2))/((s + 2)^2 + 9)^2 - 1/((s + 2)^2 + 9)
```

1- d

```
syms x t X
x=(1-exp(-2*t))^2;
X=laplace(x)
```

```
ans: 1/(s + 4) - 2/(s + 2) + 1/s
```

1- e

```
syms x t X
x=(cos(3*t))^2;
X=laplace(x)
```

```
ans: (s^2 + 18)/(s*(s^2 + 36))
```

3-a

```
syms s x X
X=2/(s^2+3*s+2);
x=ilaplace(X)
```

```
ans: 2*exp(-t) - 2*exp(-2*t)
```

3- b

```
syms s x X
X=(3*s+5)/(s^2+7);
x=ilaplace(X)
```

```
ans: 3*cos(7^(1/2)*t) + (5*7^(1/2)*sin(7^(1/2)*t))/7
```

3- c

```
syms s x X
X=5/(s^2-9);
x=ilaplace(X)
```

```
ans: (5*exp(3*t))/6 - (5*exp(-3*t))/6
```

3- d

```
syms s x X
X=3*(s+3)/(s^2+6*s+8);
x=ilaplace(X)
```

```
ans: (3*exp(-2*t))/2 + (3*exp(-4*t))/2
```

3- e

```
syms s x X
X=(s+1)/(s^4+4*s^3+4*s^2);
x=ilaplace(X)
```

```
ans: t/4 - (t*exp(-2*t))/4
```